Corrigendum to "Trading and Information Diffusion in Over-the-Counter Markets"

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As it was noticed by Yilin Wang, in our paper Babus and Kondor (2018) the first-order condition (10) is not fully consistent with our description of the OTC game. In particular, we derive the first order condition for dealer i from problem

$$\max_{(q_{ij}^i)_{j \in g^i}} \sum_{j \in g^i} q_{ij}^i \left(E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) + \frac{1}{\beta_{ij} + t_{ij}^j \left(z_{ij,ij}^j - 1\right)} q_{ij}^i - I_{ij}^j \right)$$

where we take the intercept I_{ij}^j defined by (8) as insensitive to the quantity q_{ij}^i.

In our derivations, we implicitly assume that each dealer i chooses his demand function when trading with a neighbor $j \in g^i$ to maximize her objective function (2) understanding that his residual demand on link ij is determined by market clearing conditions

$$Q_{ij}^i \left(s^i, p_{g^i}\right) + Q_{ij}^j \left(s^j, p_{g^j}\right) + \beta_{ij} p_{ij} = 0,$$

but taking prices at which she does not trade as given.

However, while dealer i does not observe prices $p_{jk, k \neq i}$, at which she does not trade, in a Bayesian Nash equilibrium dealer i should still consider the indirect effect of her quantity q_{ij}^i on p_{ij} through p_{jk}, leading to the first order condition with respect to q_{ij}^i

$$\left(E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) - p_{ij}\right) - q_{ij}^i \frac{\partial p_{ij}}{\partial q_{ij}^i} - \sum_{\substack{k \in g^i \\ k \neq j}} q_{ik}^i \frac{\partial p_{ik}}{\partial q_{ij}^i} = 0. \tag{C1}$$

In this corrigendum, we proceed as follows. In Section 1, we amend the OTC game in Babus and Kondor (2018) to restore consistency. Under this amendment, all our results and proofs remain unchanged. This is our preferred correction. In addition, in Section 2, we work out under the original specification (1) the analytical solution for the star network and (2) the general algorithm to calculate all equilibrium objects numerically in any network to account for indirect price effects. The analytical solutions for the star can be useful for applications where the star network is a reasonable starting point. We show that in this case the indirect price effects leave our qualitative results virtually unchanged. The general algorithm can be useful to compare a wide range of networks numerically, if the original specification is preferred. Based on this algorithm, we regenerate Figure 2 of the original paper for comparison. While the new and original figures are both quantitatively and qualitatively similar, we point out some new insights to inform future research.

1 The modified OTC game: Dealers as groups of traders

We amend the OTC game in Babus and Kondor (2018) as follows. As before, consider n dealers organized in a dealer network g. Let g^i denote the set of i's neighbors and $m^i \equiv |g^i|$ the number of i's neighbors. Unlike in our original set-up, we consider that each dealer i represents a group of many risk-neutral traders who have the same valuation for the asset, θ^i, and observe the same private signal, s^i, as well as any information available to the group (i.e. prices).[1] In particular, there is a mass m^i of traders affiliated with dealer i. A group i is divided into m^i unit-mass subgroups, and each subgroup is assigned a single link on which to trade. Trade is still bilateral and takes places between pairs of traders. Thus, a link between i and j indicates that a trader affiliated with dealer i and assigned to link ij can potentially enter a transaction with a trader affiliated with dealer j and assigned to link ij.

We argue that this is a sensible representation of many OTC markets. Large dealers, like the trading desks of major investment banks employ a large number of traders. While traders share their market insights, they trade independently. Usually, a significant part of their compensation is related to their own trading performance. Given the large number of these traders, it is reasonable to assume — similarly to the approach of Atkeson, Eisfeldt, and Weill (2015) — that the quantity an individual trader trades has insignificant price impact to other agents' transaction prices.

The set-up we propose here still captures the critical features of OTC markets that we emphasized in our original paper, such as "transactions are bilateral, prices are dispersed, trading relationships are persistent, and typically, a few large dealers intermediate a large share of the trading volume." Likewise, we can shed light on our original research question of "how decentralization (characterized by the structure of the dealer network) and adverse selection jointly influence information diffusion,

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[1] The joint distribution of all random variables are as in Babus and Kondor (2018).

expected profits, trading costs, and welfare." Effectively, under the modified specification dealers are the relevant units of independent information, and traders are the relevant units of independent trading. Perhaps the only limitation of the modified set-up relative to the original specification is that one of our insights that "information diffusion through prices is not affected by strategic considerations" is much less surprising.

We show that in the set-up we propose here all the results and proofs in Babus and Kondor (2018) go through without any change. To see this formally, we introduce some new notation. Within each subgroup in a group i, we index traders by $\tau \in [0, 1]$. Thus, a particular trader is identified by its group (i.e. the dealer), the link at which she trades at, and her index in the subgroup, (i, ij, τ), where $j \in g^i$. Without loss of generality, we assume that the counterparty of trader (i, ij, τ) is trader (j, ij, τ). These two traders trade at price $\hat{p}_{ij}(\tau)$. Let the trading strategy of trader (i, ij, τ) be a demand function

$$Q_{ij,\tau}^i \left(s^i, \mathbf{p}_{g^i}, \hat{p}_{ij}(\tau) \right) \tag{C2}$$

which maps the signal of the group, s^i, the vector of average prices, $\mathbf{p}_{g^i}$, that prevail in the group's transactions and the transaction price $\hat{p}_{ij}(\tau)$ into a traded quantity. Let us denote this quantity by $\hat{q}_{ij}^i(\tau)$. In particular, elements of $\mathbf{p}_{g^i}$ are defined as

$$p_{ik} \equiv \int_0^1 \hat{p}_{ik}(\tau) \, d\tau$$

for all $k \in g^i$. Trader (j, ij, τ) chooses a strategy to maximize her expected profit

$$E \left(Q_{ij,\tau}^i \left(s^i, \mathbf{p}_{g^i}, \hat{p}_{ij}(\tau) \right) \left(\theta^i - \hat{p}_{ij}(\tau) \right) | s^i, \mathbf{p}_{g^i, k \neq j}, \hat{p}_{ij}(\tau) \right)$$

where $\hat{p}_{ij}(\tau)$ is a function which is continuous almost everywhere, and defined as the solution of

$$Q_{ij,\tau}^i \left(s^i, \mathbf{p}_{g^i}, \hat{p}_{ij}(\tau) \right) + Q_{ij,\tau}^j \left(s^j, \mathbf{p}_{g^j}, \hat{p}_{ij}(\tau) \right) + \beta_{ij} \hat{p}_{ij}(\tau) = 0$$

for every link ij and index τ.[2] Just as in Babus and Kondor (2018), β_{ij} corresponds to the representative share of customers at the given link.

Then, Babus and Kondor (2018) finds a symmetric Linear Bayesian Nash equilibrium of this game where (1) $\hat{q}_{ij}^i(\tau) \equiv q_{ij}^i$, $\hat{p}_{ij}(\tau) \equiv p_{ij}$ are invariant in τ, (2) $Q_{ij,\tau}^i \left(s^i, \mathbf{p}_{g^i}, \hat{p}_{ij}(\tau) \right)$ are invariant in τ, hence have the form of $Q_{ij}^i \left(s^i, \mathbf{p}_{g^i} \right)$, and (3) $\mathbf{Q}^i(s^i; \mathbf{p}_{g^i})$ is the collection of functions $Q_{ij}^i \left(s^i, \mathbf{p}_{g^i} \right)$.

To see that with this modification the analysis in Babus and Kondor (2018) remains intact and the indirect price effect disappears, note that the impact of any dealer (i, ij, τ) on the average price p_{ij} is infinitesimal and only that average price affects other dealers' beliefs. Therefore, under this modification, Proposition 1-9 are all hold virtually unaffected.

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[2]If such $\hat{p}_{ij}(\tau)$ does not exist for all links and index, just as in the paper, we consider that markets break down and assign zero utility to all players. If there is more than one such group of functions, we choose by an arbitrary selection mechanism.

2 The indirect price effect in the original specification

In this section, we return to analyzing the Linear Bayesian Nash equilibrium of the original game in Babus and Kondor (2018). First, we fully solve the case of the n-star network and argue that in this example, the indirect price effects have minimal consequences. Second, we provide a solution method to find the equilibrium for any network by solving a nested fixed-point problem numerically. Based on this, we regenerate Figure 2 of the original paper.

2.1 The case of the $n-$star

We analyze the consequences of the indirect price effects in our main example, the $n-$star network. For this network, we can still derive the equilibrium in simple closed form expressions. We use the same notation as in Appendix B of Babus and Kondor (2018), and consider that dealer 1 is the centre, and dealers $2, ..., n$ are the periphery. All our expressions in Appendix B remain intact, except the last three which modify to

$$z_C = \frac{\bar{z}_C 2}{1 + (n-2)\bar{z}_C (1 - \bar{z}_P)} \tag{C3}$$

$$y_C = \bar{y}_C \left(1 - \frac{1}{2} \frac{z_C (n-1)(1 - \bar{z}_P)(2 - (n-2) z_C)}{2 - (n-2) z_C - \bar{z}_P z_C} \right) \tag{C4}$$

$$y_P = \bar{y}_P \left(1 - \bar{z}_P \frac{2 - (n-1) z_C}{2 - (n-2) z_C - \bar{z}_P z_C} \right). \tag{C5}$$

To see this, without loss of generality, we first derive the total price effect of periphery dealer n by solving the equation system

$$q_{n1}^n + t_C \left(y_C s^1 + \Sigma_{k=2}^n z_C p_{1k} - p_{1n} \right) + \beta p_{1n} = 0$$

$$t_P \left(y_P s^j + z_P p_{1j} - p_{1j} \right) + t_C \left(y_C s^1 + \Sigma_{k=2}^n z_C p_{1k} - p_{1j} \right) + \beta p_{1j} = 0.$$

for each $j = 2..n - 1$. The first equation gives

$$\frac{q_{n1}^n + t_C \left(y_C s^1 + \Sigma_{j=2}^{n-1} z_C p_{1j} \right)}{t_C (1 - z_C) - \beta} = p_{1n} \tag{C6}$$

while summing up the rest of the equations gives

$$\frac{t_P y_P \Sigma_{j=2}^{n-1} s^j + t_C \left((n-2) y_C s^1 + (n-2) z_C p_{1n} \right)}{-\beta + t_C (1 - (n-2) z_C) + t_P (1 - z_P)} = \Sigma_{j=2}^{n-1} p_{1j}. \tag{C7}$$

Combining (C6)-(C7), we obtain that the inverse residual demand curve is given by

$$p_{1n} = I_{n1}^1 + \lambda_{n1}^1 q_{n1}^n$$

with

$$\lambda_{n1}^1 = \frac{1}{t_C (1 - z_C) - \beta} \frac{1}{\left(1 - \frac{(z_C)^2 (t_C)^2 (n-2)}{(-\beta + t_C (1 - (n-2) z_C) + t_P (1 - z_P))(t_C (1 - z_C) - \beta)} \right)}$$

Then, the first order condition for periphery dealer n modifies to

$$q_{n1}^n =$$
$$= \left((t_C (1 - z_C) - \beta) - \frac{(t_C (1 - z_C) - \beta) (z_C)^2 (t_C)^2 (n - 2)}{(-\beta + t_C (1 - (n - 2) z_C) + t_P (1 - z_P)) (t_C (1 - z_C) - \beta)} \right) (E(\theta^n | s^n, p_{1n}) - p_{1n}).$$

(C8)

The form of the first order condition of the central agent still implies (12). Then, solving for t_C and t_P, we get

$$t_P = -2\beta \frac{(n - 1) z_C - 2}{n z_P z_C - 2 z_P - 2 z_C} \tag{C9}$$

$$t_C = -\beta (2 - z_P) \frac{(n - 2) z_C - 2}{n z_P z_C - 2 z_P - 2 z_C} \tag{C10}$$

which need to be compared to equation (22) in the paper.

Importantly, (C8) shows that, even when we account for the indirect price effect, the demand function of dealer j still has the form of $t_P \left(E \left(\theta | s^j, p_{1j} \right) - p_{1j} \right)$. This implies that the counterparty of agent j can learn the posterior of agent j from the market clearing price. Similarly, any periphery dealer j can learn the belief of the central dealer from the market clearing price. This critical property allows us to follow Proposition 2 and use the conditional guessing game to derive the equilibrium. In the particular case of the n-star, instead of system (21), we use $\bar{z}_P = \frac{1}{2} z_P$ from Appendix B and express $\bar{z}_C, \bar{y}_P, \bar{y}_C$ from (C3)-(C5). Given these equations, following the proof of Proposition 2 we show that choosing the prices and demand functions (23) and (24) in Babus and Kondor (2018) is an equilibrium of the OTC game in the n-star network, where the trading intensities are given by (C9)-(C10).

Note also, that the general observations in Section 5.1.1 of Babus and Kondor (2018) relied only on this property and that (22) implies $\frac{\partial t_{ij}^i}{\partial z_{ij}^j} < 0$, which still holds under (C9)-(C10). We have also checked that the other statements concerning the n-star network, the second part of Proposition 3 and the first part of Proposition 9 still holds unchanged.[3] That is, the effect of the indirect price effect on the equilibrium analysis is negligible for the $n-$star network.

2.2 General networks

In general networks, we show that the equilibrium can be derived by solving a nested fixed point problem which we describe in this section.

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[3] As for the second part of Proposition 9, while z_C, z_P, y_C, y_P converge to the same values in the limit $\rho \to 1$ as in Babus and Kondor (2018), because of the changing expressions of (C9)-(C10), the limits of t_C and t_P change to $t_C \to -\frac{\beta}{n}$ and $t_P \to -(n-1)\frac{\beta}{n}$. In this limit, each dealer's profit is bounded away from zero. Details are available on request.

2.2.1 Equilibrium conditions

As a first step, define Λ^i, a $\left|g^i\right| \times \left|g^i\right|$ matrix of price impacts. Λ^i has rows $\{ij\}_{j\in g^i}$ and columns $\{ik\}_{k\in g^i}$ and elements $\frac{\partial p_{ik}}{\partial q_{ij}^i} = \lambda_{ij}^{ik}$. Note that while the upper index is interchangeable, or $\lambda_{ij}^{ik} = \lambda_{ij}^{ki}$, the lower index is not, or $\lambda_{ij}^{ik} \neq \lambda_{ji}^{ik}$. Then, we rewrite (C1) as the generalized demand function for dealer i

$$\mathbf{Q}_{g^i} = T^i \left(E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) \cdot \mathbf{1}_{g^i} - \mathbf{p}_{g^i} \right) \tag{C11}$$

where $\mathbf{1}_{g^i}$ to be a $\left|g^i\right| \times 1$ column vector of $1-$s and $T^i \equiv \left(\Lambda^i\right)^{-1}$, is the inverse price impact matrix. The matrix T^i has rows $\{ij\}_{j\in g^i}$ and columns $\{ik\}_{k\in g^i}$, and we denote the element in row ij and column ik by t_{ij}^{ik}. Now, we can write all bilateral market clearing conditions in the form of

$$\sum_{i\in N} \tilde{T}^i \left(E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) \cdot \mathbf{1}_g - \tilde{\mathbf{p}} \right) + \beta \tilde{\mathbf{p}} = \mathbf{0}, \tag{C12}$$

where $\mathbf{1}_g$ is a $|g| \times 1$ column vector of $1-$s, and $\tilde{T}^i$ and $\tilde{\mathbf{p}}$ are constructed as follows. Let the links in the network g be ordered lexicographically. For each dealer i we construct $\tilde{T}^i$, a $|g| \times |g|$ expanded matrix, which has rows $\{ii'\}_{ii'\in g, i<i'}$ and columns $\{jj'\}_{jj'\in g, j<j'}$. The matrix $\tilde{T}^i$ has elements t_{ij}^{ik} in row ij (if $i < j$) or ji (if $j < i$) and column ik (if $i < k$) or ki (if $k < i$) for each $j,k \in g^i$, and 0 otherwise. Similarly, $\tilde{\mathbf{p}}$ is a column vector of all prices with price p_{ij} in row ij.

Finally, we can write the vector of conditional expectations as

$$E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) \cdot \mathbf{1}_g = y^i s^i \cdot \mathbf{1}_g + \tilde{Z}^i \cdot \tilde{\mathbf{p}} \tag{C13}$$

where $\tilde{Z}^i$ is a $|g| \times |g|$ matrix with elements z_{ij}^i in (every row of) column ij (if $i < j$) or ji (if $j < i$), for all $j \in g^i$, and 0 otherwise. Substituting (C13) into (C12), solving for $\tilde{\mathbf{p}}$, and equate the matrix expression for the inverse price impact function with T^i gives the fixed point condition

$$T^i = \left[\left[\left(-\sum_{\substack{j\in N \\ j\neq i}} \tilde{T}^j \left(\tilde{Z}^j - I \right) - \beta I \right)^{-1} \right]_{g^i} \right]^{-1} \tag{C14}$$

where I is the identity matrix of size $|g| \times |g|$ and the operator $[A]_{g^i}$ "reduces" matrix $|g| \times |g|$ A to a matrix $\left|g^i\right| \times \left|g^i\right|$ by selecting only those elements that are located in rows ij with $j \in g^i$ and columns ik with $k \in g^i$. [4]

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[4]For instance, in the case of a 3-star network where dealer 1 is the central agent, expression (C14) gives equations

$$\begin{pmatrix} t_C & 0 \\ 0 & t_C \end{pmatrix} = \begin{pmatrix} -(\beta - t_P + t_P z_P) & 0 \\ 0 & -(\beta - t_P + t_P z_P) \end{pmatrix}$$

$$t_P = \left(\left[\left[-\left(\begin{pmatrix} t_C & 0 \\ 0 & t_C \end{pmatrix} \left(\begin{pmatrix} z_C & z_C \\ z_C & z_C \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) + \begin{pmatrix} 0 & 0 \\ 0 & t_P \end{pmatrix} \left(\begin{pmatrix} 0 & z_P \\ 0 & z_P \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) + \beta \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) \right]^{-1} \right]_{g^2} \right)^{-1}$$

for dealer 1 and 2. The operator $[]_{g^2}$ selects the 11 elements of the matrix, leading to an equation system with solution (C9)-(C10).

2.2.2 The nested fixed point problem

Based on the equilibrium conditions (C12)-(C14), we can construct the following algorithm to find the equilibrium for any networks.[5]

1. For each agent i, conjecture values y^i and z_{ij}^i for all $j \in g^i$. Values z_{ij}^i define the matrix $\tilde{Z}^i$ for each i.

2. Given $\tilde{Z}^i$, find the elements of matrix T^i for each i by the fixed point problem (C14). These elements also define $\tilde{T}^i$.

3. Then, matrices $\tilde{Z}^i$ and $\tilde{T}^i$ along with the conjectured y^i values give the coefficients of each signal in $\tilde{\mathbf{p}}$. As $\tilde{\mathbf{p}}$, the signals, s^i, and values, θ^i, are jointly normally distributed, the projection theorem implies new values for y^i and z_{ij}^i by (C13). In an equilibrium, the initial conjecture and the implied y^i, z_{ij}^i coefficients should coincide.

2.2.3 Circulants, the complete network and the centralized market

Using the general algorithm above, we regenerate Figure 2 of the original paper as Figure 1. For easier comparison, we fix the scale of each panel as in Figure 2.[6]

Comparing the two figures shows that the main equilibrium objects are similar both in a qualitative and quantitative sense. However, there are some new insights which can be informative for future research. First, the comparison of the OTC market with the centralized market differs. As we see on Panels A-D, the centralized market leads to higher expected profit for traders, higher utility for customers, higher total welfare and lower illiquidity for the traders than any of the circulant networks including the complete network. This was not always the case without taking into account the indirect price effect (see Proposition 8 in Babus and Kondor (2018)). Second, related to our remark in footnote 3, the expected profit of periphery agents in a star is lower than those of the central agents even when ρ is close to 1. Whether these observations hold in general or only for our particular parameters is left for future research.

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[5] Note this solution method does not rely on a connection between the OTC game and the conditional guessing game. Hence, while the results in Babus and Kondor (2018) with respect to the conditional guessing game which are the basis of Proposition 1, 5, 6, and 7 continue to hold, their implications on the OTC game equilibrium – along with the statement of a close connection in Proposition 4 – become uncertain. Also, the existence conditions in Proposition 2 are to be replaced by the existence conditions for this fixed point. We did not investigate the existence for circular networks of the first part of Proposition 3 separately from the general case.

[6] In the original set up we defined illiquidity for agent i as the average of $\frac{1}{t_{ij}^i}$. To keep the objects as comparable as possible across the two versions, here we define it as the average of the diagonal elements in $\left(T^i\right)^{-1} = \Lambda^i$.
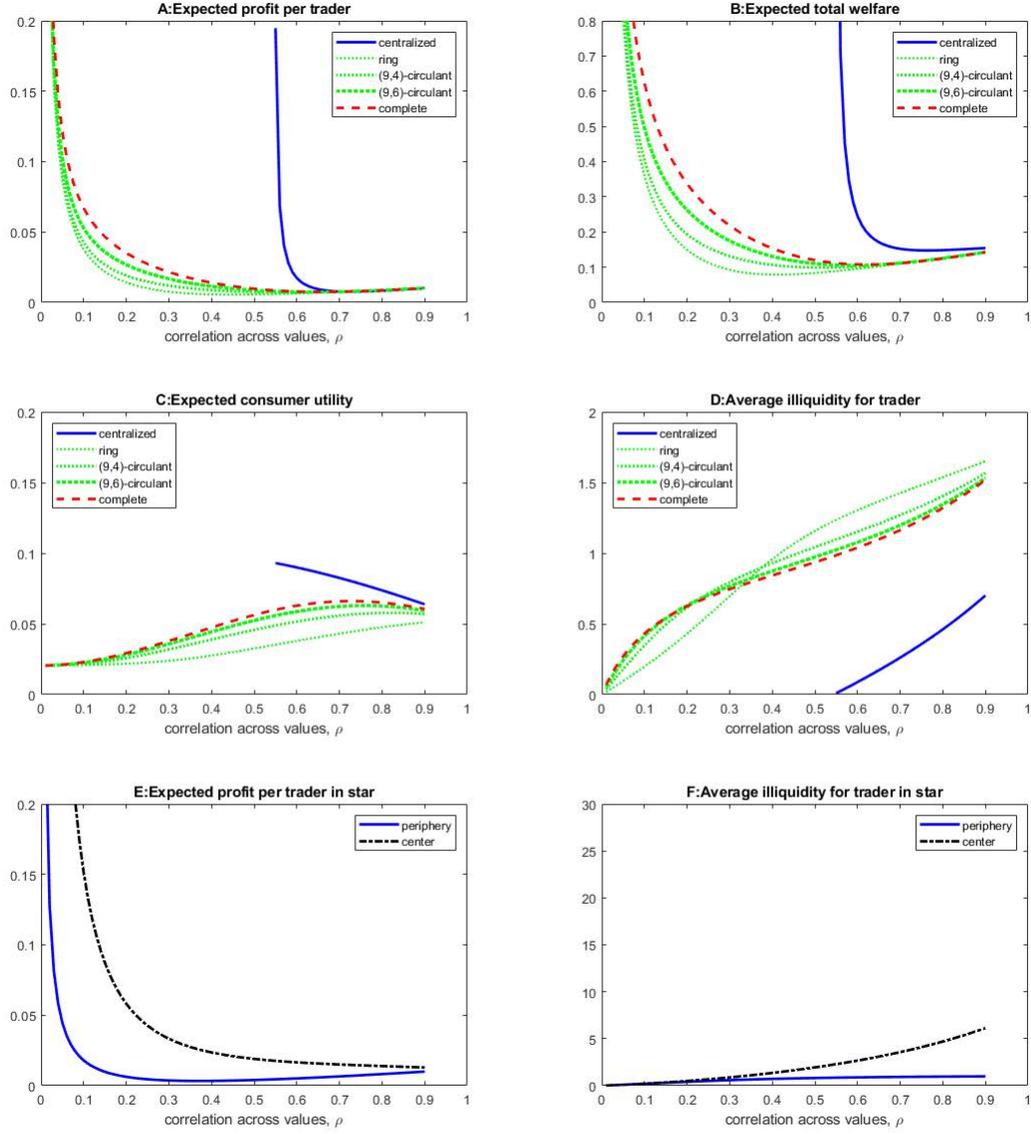
Figure 1: Expected profit, expected welfare, expected customer utility, average trading cost (illiquidity) per trader in various networks. Parameters: $n = 9$, $B = 1$, $\sigma_\theta^2 = 0.1$, $\sigma_\varepsilon^2 = 1$.

3 Conclusion

In this note, we introduced a modification of the OTC game in Babus and Kondor (2018) which restores the consitency of all the results and proofs despite the overlooked indirect price effect under the original specification. This is our preferred correction. However, for interested readers, we have also worked out the modified expressions for the $n-$star network and a general algorithm to find the equilibrium in any network under the original specification.

References

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