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SUPPLEMENT TO "REPUTATIONAL BARGAINING AND DEADLINES": THE DELAYED IMPLEMENTATION MODEL

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This supplement provides material to accompany the main text. I show how the model and results in the paper's main text extend to a hard deadline setting, when agreed deals are subject to stochastic delay in implementation.

I OUTLINE BELOW A MODEL in which there is a known hard deadline at time T, but agreements take an unknown time to implement. Transfer deadline days in professional sports offer a motivating example. Even after two teams agree a transfer before a midnight deadline, the deal may fall through if either team fails to complete the necessary paperwork in time. The model is closely related to the one in the main text, and is formally identical when $r_i = r_j = 0$.

After agreement on the principles of a deal at time t, the time it takes to implement a deal is distributed according to G on the interval $[t, T+t]$. A deal is implemented at time $t + w$ given agreement at t with continuous density $g(T - w)$. I again assume $G(t) < 1$ for $t < T$. The probability of missing the deadline of T after an agreement at t is then

$$1 - \int_0^{T-t} g(T-w)\, dw = G(t).$$

Agent i's conditional expected utility from a deal agreed at time t that gives her $\alpha_i(t)$ of the dollar is

$$\hat{u}_i\big(t, \alpha_i(t)\big) := u_i\big(\alpha_i(t)\big) \int_0^{T-t} g(T-w) e^{-r_i(w+t)}\, dw + e^{-r_i T} G(t) u_i(d_i).$$

The bargaining protocol is as in the general model of Section 6 of the main text; however, to keep things transparent, consider only simple and complex behavioral types (not very complex types). Details about such types match those in the main text; in particular, a complex type's demands are continuously differentiable and satisfy $1 - \alpha_i(T) > d_j$. It is optimal to concede against a known complex type immediately. Given this, Lemma 3 goes through without change; an agent who is known to be rational must concede immediately to an agent who might be a complex type. Rational agents' strategies, therefore, reduce to choosing a behavioral type to mimic (described by μ_1 and $\mu_2^{\alpha_1}$) and a concession time (described by F_i^{α}). Given demands and agent j's strategy, agent i's expected utility from concession at time t (assuming no probabilistic concession by j at t) is

$$U_i(t, \sigma_j | \alpha) = \int_0^t \hat{u}_i\big(s, \alpha_i(s)\big)\, dF_j(s) + \big(1 - F_j(t)\big) \hat{u}_i\big(t, 1 - \alpha_j(t)\big).$$

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The arguments of Lemma 2 again go through immediately. In particular, agents must be indifferent to conceding on $(0, \tau^*]$ and reach a probability 1 reputation at τ^*. This implies that agent j's concession rate is

$$\begin{aligned}\frac{f_j(t)}{1-F_j(t)} &= \lambda_j(t)\\ &:= \Biggl(r_i u_i\bigl(1-\alpha_i(t)\bigr)\\ &\qquad + \frac{e^{-r_i T} g(t)}{\displaystyle\int_0^{T-t} g(T-w)e^{-r_i(w+t)}\,dw}\\ &\qquad \times \bigl(u_i\bigl(1-\alpha_j(t)\bigr)-u_i(d_i)\bigr)+\alpha_j'(t)u_i'\bigl(1-\alpha_j(t)\bigr)\Biggr)\\ &\qquad \Big/\bigl(u_i\bigl(\alpha_i(t)\bigr)-u_i\bigl(1-\alpha_j(t)\bigr)\bigr).\end{aligned}$$

The probability of concession up to time t conditional on no time zero concession is, therefore, $\hat{F}_j(t) = \exp(\int_0^t \lambda_j(s)\,ds)$. Exhaustion times and time zero concession are defined exactly as before, characterizing the unique equilibrium. Notice that

$$\int_0^{T-t} g(T-w)e^{-r_i(w+t)}\,dw \in \bigl[e^{-r_i T}\bigl(1-G(t)\bigr), 1-G(t)\bigr].$$

This means that as r_i becomes arbitrarily small, $1-\hat{F}_j(t)$ becomes arbitrarily close to $(1-G(t))^{K_j^g}$ where K_j^g is as defined in the paper. This bound ensures that Proposition 3 and Lemma 4 from the main text (the deadline effects results) go through immediately and unchanged. This bound also implies that Proposition 4 (convergence to the Nash solution given simple types only) goes through immediately and without change.

The (generalized) Rubinstein bargaining demand in this setting is defined by the ODE:

$$\begin{aligned}2\alpha_i'^{\mathrm{R}}(t) &= \frac{r_i u_i\bigl(\alpha_i^{\mathrm{R}}(t)\bigr)}{u_i'\bigl(\alpha_i^{\mathrm{R}}(t)\bigr)}\\ &\quad + \frac{e^{-r_i T} g(t)}{\displaystyle\int_0^{T-t} g(T-w)e^{-r_i(w+t)}\,dw}\frac{u_i\bigl(1-\alpha_i^{\mathrm{R}}(t)\bigr)-u_i(d_i)}{u_i'\bigl(\alpha_i^{\mathrm{R}}(t)\bigr)}\end{aligned}$$

$$- \frac{r_j u_j\big(1-\alpha_i^{\mathrm{R}}(t)\big)}{u_j'\big(1-\alpha_i^{\mathrm{R}}(t)\big)} - \frac{e^{-r_j T} g(t)}{\displaystyle\int_0^{T-t} g(T-w) e^{-r_j(w+t)}\,dw} \frac{u_j\big(1-\alpha_i^{\mathrm{R}}(t)\big) - u_j(d_j)}{u_j'\big(1-\alpha_i^{\mathrm{R}}(t)\big)}.$$

Using the same trick as in the text of defining $l_i(t) = u_i(\alpha_i(t)) - u_i(1-\alpha_j(t))$, the proof of Proposition 5 (convergence to the generalized Rubinstein solution with complex types) works exactly as before. Let $\check{T}$ be defined as in the main text, so that either $l_i(\check{T}) = 0$ or $G(\check{T}) = 1$. Suppose that $z_{i,n} \to 0$, agent 1 demands more than the initial offer of some Below Rubinstein type of agent 2 with positive limit probability, and agent 2 always makes the Below Rubinstein counterdemand. Following such demands we must have $\bar{z}_{i,n} \to 0$, $\frac{\bar{z}_{1,n}}{\bar{z}_{2,n}} < L_2$ for some positive constant L_2, and therefore $\tau^* \to \check{T}$ by the same arguments as in Proposition 4. Again, using familiar arguments, time zero concession must satisfy

$$\begin{aligned} \frac{c_1}{c_2} &= \frac{\bar{z}_1}{\bar{z}_2} \exp\left(\int_0^{\tau^*} \lambda_1(s) - \lambda_2(s)\,ds\right) \\ &\leq \frac{\bar{z}_1}{\bar{z}_2} \frac{l_2(\tau^*)}{l_2(0)} \frac{\displaystyle\int_0^{T-\tau^*} g(T-w) e^{-r_2(w+\tau^*)}\,dw}{\displaystyle\int_0^{T} g(T-w) e^{-r_2 w}\,dw} \\ &\leq L_2 \frac{l_2(\tau^*)}{l_2(0)} e^{r_2 T}\big(1 - G(\tau^*)\big), \end{aligned}$$

where the right hand side must converge to zero as $\tau^* \to \check{T}$.

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