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SUPPLEMENT TO "SHARING RULE IDENTIFICATION FOR GENERAL COLLECTIVE CONSUMPTION MODELS"

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DEMAND SYSTEM SPECIFICATIONS

THREE OF THE FOUR DEMAND SYSTEMS IN THE PAPER are based on Banks, Blundell, and Lewbel's (1997) Quadratic Almost Ideal Demand System, a parametric demand system that is a so-called flexible functional form. The fourth system is an entirely nonparametric one. To make this supplement self-contained, we will repeat some of the discussion contained in the main paper.

S.1. *QUAIDS Version 1: Without SR1 and Without Taste Shifters*

The first version of the parametric demand system in our paper is QUAIDS without SR1 imposed and without any taste shifters (see column RP2 in Table 3 of the paper). Denote the budget share of commodity i ($i = 1, \dots, 5$) by w_i, full income by y, and the vector of prices by $\mathbf{p} = (p_1, \dots, p_5)'$. Our first version of QUAIDS corresponds to the equation

$$w_i = \alpha_i + \beta_i \ln\left[\frac{y}{a(\mathbf{p})}\right] + \frac{\lambda_i}{b(\mathbf{p})}\left\{\ln\left[\frac{y}{a(\mathbf{p})}\right]\right\}^2 + \sum_{j=1}^{5} \gamma_{ij} \ln p_j, \tag{S.1}$$

where

$$\ln a(\mathbf{p}) = \alpha_0 + \sum_{i=1}^{5} \alpha_i \ln p_i + \frac{1}{2}\sum_{i=1}^{5}\sum_{j=1}^{5} \gamma_{ij} \ln p_i \ln p_j,$$

$$b(\mathbf{p}) = \prod_{i=1}^{5} p_i^{\beta_i},$$

$$\lambda(\mathbf{p}) = \sum_{i=1}^{5} \lambda_i \ln p_i.$$

The parameters α_i, β_i, λ_i, and γ_{ij} ($\forall i, j$) must be estimated. Adding-up implies that $\sum_i \alpha_i = 1$, $\sum_i \beta_i = 0$, $\sum_i \lambda_i = 0$, and $\sum_i \gamma_{ij} = 0$ ($\forall j$), while homogeneity requires $\sum_j \gamma_{ij} = 0$ ($\forall i$). Adding-up is then satisfied and, as a result, we only need to estimate four out of the five demand equations. Via the above restrictions, we can obtain the parameters of the demand equation that is left out.

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Homogeneity is imposed by estimating the system in terms of deflated prices and deflated full income. Following Banks, Blundell, and Lewbel (1997), the parameter α_0 is set to a value just below the lowest value of $\ln y$ observed in the data.

Note that, although there are no taste shifters included, we do account for full heterogeneity across couples with and without children, by estimating the system twice and independently of each other: one time on the sample of couples without children, and a second time on the sample of couples with children. Consequently, all parameters of the system can be different across both samples. Assuming additive errors and the (standard) exogeneity of wages, prices, and full income, we can obtain estimates of the QUAIDS parameters by means of multiple equation nonlinear least squares. Note that this can be shown to be a GMM estimator.

Denote the observed budget share for commodity i of observation h by $w_{i,h}$, while the corresponding estimated budget share through equation (S.1) is denoted by $\widehat{w}_{i,h}(\boldsymbol{\varphi})$, where $\boldsymbol{\varphi}$ is a vector that contains the free parameters of the system. The stacked vector of error terms of household h is given by $\mathbf{u}_h = (u_{1,h}, \dots, u_{4,h})'$, where $u_{i,h} = w_{i,h} - \widehat{w}_{i,h}(\boldsymbol{\varphi})$, while $\mathbf{u} = (\mathbf{u}_1', \dots, \mathbf{u}_H')'$, where H is the number of households in the sample. Estimates of the unknown parameters are obtained by solving the minimization problem $\min_{\boldsymbol{\varphi}} \mathbf{u}'\mathbf{u}$. Starting values for the minimization procedure are obtained through the estimation of a linearized version of QUAIDS. The latter is obtained by setting the price index $b(\mathbf{p})$ equal to 1 in equation (S.1) and by substituting the Stone index, defined as $\sum_j \overline{w}_j \log p_j$ (where $\overline{w}_j$ is the average budget share of commodity j), for the price index $a(\mathbf{p})$. Standard errors are obtained by a bootstrap procedure. Table S.I shows the parameter estimates and their standard errors for both samples (couples with and without children). Note that we only show the effectively estimated parameters of QUAIDS. The remainder of the parameters can be calculated by means of the adding-up and homogeneity restrictions. Commodity 1 is the husband's leisure, commodity 2 the wife's leisure, commodity 3 food, commodity 4 housing, and commodity 5 is other goods. The demand equation of commodity 5 is left out of the system due to adding-up.

S.2. *QUAIDS Version 2: With SR1 but Without Taste Shifters*

The second version of the parametric demand system in our paper is QUAIDS with SR1 imposed but without any taste shifters (see column RP3 in Table III of the paper). Similarly to before, we allow for heterogeneity across couples with and without children by separately estimating the model parameters on the two different samples. The second version of QUAIDS differs from the first version in the sense that we now impose Browning and Chiappori's (1998) SR1 condition, which implies that the pseudo-Slutsky matrix can be decomposed into the sum of a symmetric negative semi-definite matrix and

TABLE S.I
QUAIDS WITHOUT SR1 AND WITHOUT TASTE SHIFTERS

	No Children		Children	
	Coef.	Std. Err.	Coef.	Std. Err.
α_1	1.1219	0.4831	0.9585	0.4680
β_1	−0.0884	0.1003	−0.0368	0.0944
λ_1	−0.0021	0.0041	−0.0058	0.0034
γ_{11}	0.0903	0.0395	0.1090	0.0516
γ_{12}	−0.2571	0.0303	−0.2194	0.0437
γ_{13}	0.1445	0.0530	0.2266	0.0397
γ_{14}	0.0366	0.1196	−0.2557	0.0610
α_2	1.0145	0.5106	0.9470	0.8077
β_2	−0.0495	0.1007	−0.0443	0.1624
λ_2	−0.0048	0.0035	−0.0037	0.0059
γ_{21}	−0.2494	0.0289	−0.2158	0.0411
γ_{22}	0.1020	0.0572	0.1232	0.1044
γ_{23}	0.1173	0.0496	0.0936	0.0365
γ_{24}	0.0311	0.0948	−0.0772	0.0756
α_3	−0.1325	0.3469	−1.0679	0.4969
β_3	0.0181	0.0864	0.2503	0.1223
λ_3	0.0002	0.0052	−0.0142	0.0072
γ_{31}	0.0091	0.0296	0.0425	0.0275
γ_{32}	0.0082	0.0243	0.0509	0.0227
γ_{33}	0.0297	0.0724	−0.2708	0.1382
γ_{34}	−0.0492	0.1911	0.4642	0.1919
α_4	−1.4743	1.3704	1.4529	0.5348
β_4	0.3598	0.3426	−0.4023	0.1391
λ_4	−0.0191	0.0206	0.0290	0.0091
γ_{41}	0.1835	0.1046	−0.0421	0.0425
γ_{42}	0.1208	0.0737	−0.0665	0.0714
γ_{43}	−0.2389	0.1965	0.2571	0.1940
γ_{44}	−0.4471	0.4581	−0.5197	0.2645

a matrix of rank 1. As discussed in the main text, the SR1 condition holds if and only if, for all i, k such that $k > i > 2$,

$$m_{ik} = \frac{m_{1i}m_{2k} - m_{1k}m_{2i}}{m_{12}}, \tag{S.2}$$

where, without loss of generality, m_{12} is assumed to be different from zero. To obtain SR1-restricted QUAIDS parameters, we estimate the parameters in the budget share equations (S.1) by means of multiple equation nonlinear least squares, while imposing the equality restrictions in (S.2). Table S.II shows the parameter estimates and their standard errors for both samples (couples with and without children) for this version of QUAIDS.

TABLE S.II

QUAIDS WITH SR1 AND WITHOUT TASTE SHIFTERS

	No Children		Children	
	Coef.	Std. Err.	Coef.	Std. Err.
α_1	1.1215	0.5723	0.7960	0.5924
β_1	−0.0885	0.1190	−0.0146	0.1195
λ_1	−0.0021	0.0047	−0.0058	0.0042
γ_{11}	0.0900	0.0430	0.1468	0.0615
γ_{12}	−0.2573	0.0340	−0.1489	0.0473
γ_{13}	0.1530	0.0512	0.1696	0.0425
γ_{14}	0.0385	0.1097	−0.2061	0.0726
α_2	1.0192	0.6137	−0.8624	0.7942
β_2	−0.0505	0.1205	0.3157	0.1664
λ_2	−0.0047	0.0040	−0.0165	0.0077
γ_{21}	−0.2505	0.0309	−0.1530	0.0442
γ_{22}	0.1013	0.0670	−0.0838	0.1079
γ_{23}	0.1122	0.0519	0.0691	0.0446
γ_{24}	0.0113	0.0834	0.0792	0.0773
α_3	−0.1438	0.3297	0.1888	0.4816
β_3	0.0210	0.0844	−0.0512	0.1181
λ_3	−0.0000	0.0053	0.0030	0.0070
γ_{31}	0.0104	0.0298	−0.0106	0.0297
γ_{32}	0.0098	0.0211	0.0321	0.0227
γ_{33}	0.0309	0.0774	0.0234	0.1334
γ_{34}	−0.0560	0.1899	−0.0375	0.2095
α_4	−1.4310	1.4612	0.5913	0.6130
β_4	0.3487	0.3623	−0.1486	0.1572
λ_4	−0.0184	0.0213	0.0098	0.0100
γ_{41}	0.1788	0.1026	0.0051	0.0613
γ_{42}	0.1208	0.0786	0.1179	0.0676
γ_{43}	−0.2409	0.1918	−0.2446	0.2068
γ_{44}	−0.4083	0.4612	0.2196	0.3139

We conducted a test to check whether the household's demand satisfies the SR1 condition. More specifically, we used the nonlinear analog of the F-statistic, which has the advantage that it is based on both the unrestricted and the restricted estimates (whereas the Wald-statistic is not invariant to how the null hypothesis is formulated). For the sample of childless couples, the test statistic equals 0.1824, while the critical value for 3 degrees of freedom in the numerator and 837 degrees of freedom for the denominator equals about 2.60. This corresponds to a p-value of 0.9084. For couples with children, the test statistic equals 0.4775, while the critical value for 3 degrees of freedom in the numerator and 1362 degrees of freedom for the denominator equals about 2.60, too. This corresponds to a p-value of 0.6980. Consequently, we cannot reject the null hypothesis that household demand satisfies the SR1 condition.

TABLE S.III
QUAIDS WITH SR1 AND TASTE SHIFTERS

	No Children		Children	
	Coef.	Std. Err.	Coef.	Std. Err.
$\alpha_{1,0}$	−0.4012	0.6970	0.8186	0.6303
$\alpha_{1,1}$	−0.0000	0.0003	−0.0000	0.0002
$\alpha_{1,2}$	−0.0094	0.0082	−0.0016	0.0050
β_1	0.2265	0.1396	−0.0192	0.1259
λ_1	−0.0142	0.0047	−0.0056	0.0044
γ_{11}	0.0474	0.0666	0.1468	0.0639
γ_{12}	−0.2540	0.0581	−0.1490	0.0469
γ_{13}	0.1455	0.0642	0.1658	0.0382
γ_{14}	0.0861	0.1068	−0.2072	0.0695
$\alpha_{2,0}$	0.1009	0.6016	−0.8159	0.6643
$\alpha_{2,1}$	0.0004	0.0002	−0.0008	0.0002
$\alpha_{2,2}$	−0.0034	0.0078	−0.0031	0.0044
β_2	0.1248	0.1193	0.3087	0.1312
λ_2	−0.0105	0.0041	−0.0158	0.0047
γ_{21}	−0.2526	0.0544	−0.1450	0.0439
γ_{22}	0.1255	0.1213	−0.0787	0.1045
γ_{23}	0.1334	0.0641	0.0538	0.0381
γ_{24}	0.0091	0.0782	0.0974	0.0567
$\alpha_{3,0}$	0.1452	0.2309	0.2139	0.3447
$\alpha_{3,1}$	−0.0002	0.0001	−0.0000	0.0001
$\alpha_{3,2}$	0.0009	0.0028	−0.0036	0.0025
β_3	−0.0411	0.0514	−0.0549	0.0825
λ_3	0.0029	0.0025	0.0032	0.0047
γ_{31}	0.0179	0.0360	−0.0112	0.0234
γ_{32}	0.0103	0.0246	0.0358	0.0193
γ_{33}	0.0214	0.0306	0.0234	0.0919
γ_{34}	−0.0473	0.0482	−0.0438	0.1432
$\alpha_{4,0}$	1.4582	0.4580	0.5876	0.4388
$\alpha_{4,1}$	0.0001	0.0002	0.0004	0.0002
$\alpha_{4,2}$	0.0066	0.0054	0.0120	0.0035
β_4	−0.3356	0.1020	−0.1527	0.1090
λ_4	0.0176	0.0050	0.0100	0.0068
γ_{41}	0.1933	0.0967	0.0009	0.0600
γ_{42}	0.1145	0.0710	0.1227	0.0555
γ_{43}	−0.2189	0.0530	−0.2280	0.1411
γ_{44}	−0.0728	0.1913	0.2022	0.2256

S.3. *QUAIDS Version 3: With SR1 and Taste Shifters*

The final parametric demand model in our paper is again based on equation (S.1), but now with taste shifters (in addition to general observed heterogeneity across couples with and without children) included, and with the above SR1 condition imposed. More specifically, two (standard) taste shifters are included: the husband's age and a dummy for homeownership. Multicollinearity issues prevented us from also including the wife's age. The above parameter α_i now equals the function $\alpha_{i,0} + \alpha_{i,1}t_1 + \alpha_{i,2}t_2$, where t_1 is the husband's age and t_2 the homeownership dummy. Table S.III shows the associated parameter estimates and standard errors for both samples (couples with and without children).

Also for this specification, we conducted an F-test to test the null hypothesis that household demand satisfies the SR1 condition. The test statistic for childless couples (resp. couples with children) equals 0.2703 (resp. 0.3368), while the critical value equals about 2.60. This implies a p-value for childless couples (couples with children) of 0.8468 (0.7987).

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