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CORRIGENDUM TO "MAXIMALITY IN THE FARSIGHTED STABLE SET"

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Lemma 1 of Ray and Vohra (2019) is false as stated, but holds under alternative conditions which are consistent with the ideas of coalitional sovereignty that motivate the cited paper.

KEYWORDS: Stable sets, farsightedness.

1. INTRODUCTION

LEMMA 1 OF RAY AND VOHRA (2019) is false as stated. This lemma is a key part of the proof of the main theorem (Theorem 1) of the cited paper.

We give an alternative condition under which Lemma 1 holds. In addition to existing restrictions, this specifies that when a coalition of players T is broken up by the participation of some of its players in a coalitional move by S, then the remainder $T \setminus S$ is part of the new coalition structure and the new payoffs for $T \setminus S$ depend on neither the coalitions and payoffs of players outside of T before the breakup, nor the coalitions and payoffs of players outside of $T \setminus S$ after the breakup, nor the identities of players in S who are not members of T. It is possible to give more general and concise conditions (Newton (2020)), but the condition given here has the benefit that a new proof is not required.

It should be noted that the alternative condition given here and the weaker conditions in Newton (2020) are wholly consistent with the discussion of the remainder problem in the earlier work of Ray and Vohra (2015):

> "For coalition sovereignty, what is important is that the deviating coalition not be allowed to choose how the residuals organize themselves or how they distribute their surplus among themselves; these decisions must be taken as exogenously given by the deviating coalition."

Section 2 reprises the relevant parts of the model, states the lemma, and shows the gap in the proof. Section 3 shows how this can be fixed.

2. MODEL

2.1. *Coalitional Games*

A coalitional game is described by a finite set of N players and a *characteristic function* V that assigns to each nonempty *coalition* $S \subseteq N$ a nonempty, closed set of feasible payoff vectors $V(S)$. Normalize so that singletons obtain zero and assume that $V(S) \cap \mathbb{R}^S_+$ is bounded.

2.2. *States and Effectivity*

A *state* is a partition π of N, along with a payoff profile u feasible for that partition. A typical state x is therefore a pair (π, u) (or $\{\pi(x), u(x)\}$ when we need to be explicit),

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where $u_S \in V(S)$ for each $S \in \pi$. Let X be the set of all states. An *effectivity correspondence* $E(\cdot, \cdot)$ specifies, for any states x and y, the collection of coalitions $E(x, y)$ that have the power to move from x to y. Assume:

(E.1) If $S \in E(x, y)$, $T \in \pi(x)$, and $T \cap S = \emptyset$, then $T \in \pi(y)$ and $u_T(x) = u_T(y)$.

(E.2) For every state x, coalition S, partition μ of S, and payoff $v \in \mathbb{R}^S$ with $v_W \in V(W)$ for each $W \in \mu$, there is $y \in X$ such that $S \in E(x, y)$, $\mu \subseteq \pi(y)$, and $u_S(y) = v$.

2.3. *Farsighted Domination*

A *chain* is a finite collection of states $\{y^0, y^1, \dots, y^m\}$ and coalitions $\{S^1, \dots, S^m\}$, such that for every $k \geq 1$, we have $y^{k-1} \neq y^k$, and $S^k \in E(y^{k-1}, y^k)$. A state y *farsightedly dominates* x if there is a chain with $y^0 = x$ and $y^m = y$ such that for all $k = 1, \dots, m$, $u_{S^k}(y) \gg u_{S^k}(y^{k-1})$. This associated chain will be called a *blocking chain*.

2.4. *The Lemma*

The proof of Theorem 1 of Ray and Vohra (2019) relies on the following lemma.

LEMMA 1: *Suppose that y farsightedly dominates x via the chain* $\{\tilde{y}^0, \tilde{y}^1, \dots, \tilde{y}^{\tilde{m}-1}, \tilde{y}^{\tilde{m}}\}$, $\{\tilde{S}^1, \dots, \tilde{S}^{\tilde{m}}\}$, *where* $\tilde{y}^0 = x$ *and* $\tilde{y}^{\tilde{m}} = y$. *Then there exists another* (*canonical*) *blocking chain* $\{y^0, y^1, \dots, y^{m-1}, y^m\}$, $\{S^1, \dots, S^m\}$ *such that*

(i) $y^0 = x$ *and* $y^m = y$;

(ii) S^i *and* S^j *are disjoint for all* $i, j = 1, \dots, m-1$, *where* $i \neq j$;

(iii) $\bigcup_{k=1}^{m-1} S^k \subseteq S^m$, *so the set of all active movers in the canonical chain is* S^m.

This lemma turns out to be false. The problem in the argument of the original paper arises in the following passage:

> "Set $y^0 = x$ and $S^1 = \tilde{S}^1$ and, if $m \geq 2$, then recursively let $S^k = \tilde{S}^k - \bigcup_{t<k} \tilde{S}^t$ for all $k = 2, \dots, \tilde{m} - 1$. For any $k = 1, \dots, \tilde{m} - 1$, when coalition S^k moves, it does so by breaking into singletons. So, for any such k, the corresponding coalition structure, π^k, is such that all players in $\bigcup_{t<k} \tilde{S}^t$ [Author: probable typo, immaterial to our current argument, $t < k$ should be $t \leq k$] are singletons, and (by Condition (E.1)) all other players belong to the same coalition as in $\tilde{y}^k$."

It turns out that the final statement does not follow from (E.1), a fact that can be exploited to construct a counterexample (Newton (2020)).

3. SUFFICIENT CONDITIONS FOR THE LEMMA

Fortunately, it is possible to impose stronger conditions so that Lemma 1 holds. Condition (E.1) only relates to $T \in \pi(x)$ that do not intersect with a coalition S moving away from x. The replacement for (E.1) also relates to $T \in \pi(x)$ that do intersect with S.

DEFINITION 1: Given $S \in E(x, y)$, a coalition T is *unaffected* if $T \in \pi(x)$ and $S \cap T = \emptyset$. A coalition T is *affected* if $T \in \pi(x)$ and $S \cap T \neq \emptyset$.

(E.1') Let $S \in E(x, y)$. If T is an unaffected coalition, then $T \in \pi(y)$ and $u_T(x) = u_T(y)$. If T is an affected coalition, then $T \setminus S \in \pi(y)$ and $u_{T\setminus S}(y)$ depends on x only via the original payoff $u_T(x)$ in the sense that, if $S' \in E(x', y')$, $T \in \pi(x')$, $u_T(x') = u_T(x)$, and $T \setminus S' = T \setminus S$, then $u_{T\setminus S}(y') = u_{T\setminus S}(y)$.

Assuming (E.1'), (E.2) and further assuming that $V(S) \in \mathbb{R}^S_+$ for all $S \subseteq N$, the construction in the cited paper works and Lemma 1 holds.

The part of (E.1') that deals with unaffected coalitions is identical to (E.1). The part that deals with affected coalitions specifies that if a coalition T is broken up due to a coalitional move by S, then the remainder $T \setminus S$ is part of the resulting partition. Furthermore, the new payoffs for $T \setminus S$ depend on neither the coalitions and payoffs of players outside of T before the breakup, nor the coalitions and payoffs of players outside of $T \setminus S$ after the breakup, nor the identities of players in S who are not members of T. Note that, concordant with the motivation given in Ray and Vohra (2019), a moving coalition S must take the resulting behavior of players outside of S as given.

Condition (E.1') leaves the remainder $T \setminus S$ as a cell of the resulting partition and is thus similar to the approach of Green (1974). It turns out that other approaches, such as requiring that the remainder break up into singletons as in Feldman (1974), are also compatible with Lemma 1. In fact, all that is required is that the coalition structure formed by the remainder in the new partition be independent of the same factors mentioned in the previous paragraph with respect to the new payoffs. For a concise formal statement of this condition and an accompanying proof of Lemma 1, the reader is referred to Newton (2020).

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