

May 5, 2015

LAST-IN FIRST-OUT OLIGOPOLY DYNAMICS: CORRIGENDUM

BY JAAP H. ABBRING AND JEFFREY R. CAMPBELL¹

In Abbring and Campbell (2010), we presented a model of dynamic oligopoly with ongoing demand uncertainty, sunk costs of entry, and per-period fixed costs that can be avoided only with irreversible exit. Incumbent firms make their survival decisions sequentially in order of their age, oldest first, and we studied *last-in first-out* Markov-perfect equilibria in which an entrant expects to produce no longer than any incumbent. We showed that there exists such an equilibrium and that it is (essentially) unique. We also provided four conditions on the stochastic process for demand (Monotonicity, Independence, Concavity and Continuity) that guarantee that demand threshold rules govern firms' equilibrium entry and exit decisions. These results are all correct. However, we also claimed that each demand process that satisfies these four conditions can be written as a particular mixture of stochastic processes, and vice versa (Proposition 5). In this corrigendum, we note that this representation theorem requires an additional restriction on the class of mixtures. We also show that this additional restriction is not needed to guarantee threshold rules in equilibrium; for this, it suffices that the demand process is a mixture as in the original (incorrect) representation theorem, even if it does not satisfy the four conditions. Finally, we also correct a closely related error in the discussion of a demand process that does not generate threshold rules.

1. THE REPRESENTATION THEOREM

To place the error in our representation theorem into context, we repeat here the four conditions on the stochastic process for demand that we impose.

ASSUMPTION 1 (Monotonicity) $\mathbb{E}[C'|C]$ is weakly increasing in C .

ASSUMPTION 2 (Independence) The innovation error $U' \equiv C' - \mathbb{E}[C'|C]$ is independent of C .

ASSUMPTION 3 (Concavity) The cumulative distribution function \tilde{Q} of U' is concave on $[\underline{U}, \infty)$, where $\underline{U} \equiv \inf_C \{C - \mathbb{E}[C'|C]\}$.

ASSUMPTION 4 (Continuity) $\mathbb{E}[C'|C]$ is continuous from the left.

¹We are grateful to Yifan Yu, whose questions prompted us to reexamine our work. The research of Jaap Abbring is financially supported by the Netherlands Organisation for Scientific Research (NWO) through Vici grant 016.125.624. The views expressed herein are those of the authors, and they do not necessarily represent those of the Federal Reserve Bank of Chicago, the Federal Reserve System, or its Board of Governors.

Here, C' represents the next period's value of C . Proposition 4 in the article uses these conditions to show that firms' entry and exit decisions can be represented with threshold rules. The first two assumptions are straightforward, and Assumption 4 is technical. Assumption 3 is the restriction with economic content. It requires that a small increase in C cannot move a "large" probability mass of C' over an entry threshold and thereby greatly reduce an incumbent's expected future profits.

Proposition 5 in the paper asserts that all such stochastic processes $\{C\}$ (characterized by their transition function $Q(c'|C) \equiv \Pr[C' \leq c'|C]$) are equivalent to mixtures of stochastic processes with certain uniformly-distributed innovations. As stated in the text, it is incorrect. Here, we present the correct version, which adds a single condition to its statement.²

PROPOSITION 5* *The transition function $Q(\cdot|C)$ satisfies Assumptions 1–3 if and only if there exist a sequence $Q^1(\cdot|C), Q^2(\cdot|C), \dots$ of transition functions such that*

$$\lim_{K \rightarrow \infty} \sup_{c, C} |Q^K(c|C) - Q(c|C)| = 0$$

and, for all $K \in \mathbb{N}$,

$$Q^K(\cdot|C) = \sum_{k=1}^K p_k^K Q_k^K(\cdot|C), \quad p_1^K, \dots, p_K^K \geq 0, \quad \sum_{k=1}^K p_k^K = 1;$$

where, for $k = 1, \dots, K$,

$$Q_k^K(c|C) = \begin{cases} 0, & \text{if } c - \mu_k^K(C) < -\sigma_k^K/2, \\ (c - \mu_k^K(C) + \sigma_k^K/2)/\sigma_k^K, & \text{if } -\sigma_k^K/2 \leq c - \mu_k^K(C) < \sigma_k^K/2, \\ 1, & \text{otherwise,} \end{cases}$$

$$\sigma_k^K \geq 0,$$

$$\mu_k^K(C) \text{ weakly increases in } C,$$

$$\hat{C} + \sigma_k^K/2 \leq \mu_k^K(C) \leq \check{C} - \sigma_k^K/2,$$

$$\mu_k^K(C) \leq C + \sigma_k^K/2,$$

and

$$(1^*) \quad \mu_k^K(C) = \nu(C) + \nu_k^K \text{ for some function } \nu \text{ and constant } \nu_k^K.$$

²The original statement of Proposition 5 also did not properly account for the possibility that some of the mixing distributions might have zero variances, and we have corrected that here as well. The proposition's proof did not suffer from this oversight.

The single additional restriction is (1^*) , which requires the difference between any two conditional means of the stochastic processes not to depend on C . A simple counterexample illustrates why the proposition would be incorrect without (1^*) . Suppose that C takes values in $[0, 1]$ and let $p_1^K = p_2^K = 1/2$, $\mu_1^K(C) = C$, $\sigma_1^K = 0$, $\mu_2^K = 1/2$, and $\sigma_1^K = 1$ for all K . Then, the innovation error U' has variance $5/48 - C(1 - C)/4$ conditional on C and so is clearly *not* independent of C .

As the counterexample suggests, the original Proposition 5 was incorrect because the mixture representation given did not imply Assumption 2. The converse statement that Assumptions 1–3 imply the given mixture representation is true.³ The original proof of this was constructive, and the construction satisfies (1^*) . Therefore, the proof of Proposition 5* requires only demonstrating the “if” part and not the “only if” part. This corrigendum’s appendix provides this proof.

Assumptions 1–4 are sufficient for an equilibrium to feature threshold-based decision rules, but they are not necessary. The demand process from the pencil-and-paper example in Section 2 of the paper illustrates this. In it, $C' = C$ with probability λ and equals a draw from a uniform distribution over $[\hat{C}, \check{C}]$ with the complementary probability. This process does not satisfy Assumption 2, but the last-in first-out Markov-perfect equilibrium strategies have a threshold representation. However, this simple process does not have the mixture-representation *originally* presented in Proposition 5, without the additional condition in (1^*) . This example suggests that the weaker requirement that the demand process has the mixture-representation as originally presented might also guarantee that the last-in first-out equilibrium strategies have a threshold representation. Inspection of our proof of Proposition 4— which used Proposition 5 to replace Assumptions 1–3 with the originally presented mixture representation— shows that this is indeed the case.⁴ For the sake of completeness, we state this conclusion here as a corollary.

COROLLARY 1 *Let (A_S, A_E) be the unique symmetric Markov-perfect equilibrium in a LIFO strategy that defaults to inactivity. If the transition function $Q(\cdot|c)$ satisfies*

$$\lim_{K \rightarrow \infty} \sup_{c, C} |Q^K(c|C) - Q(c|C)| = 0$$

where for all $K \in \mathbb{N}$,

$$Q^K(\cdot|C) = \sum_{k=1}^K p_k^K Q_k^K(\cdot|C), \quad p_1^K, \dots, p_K^K \geq 0, \quad \sum_{k=1}^K p_k^K = 1;$$

³This is fortunate, because the proof of Proposition 4 uses Proposition 5 to replace these three assumptions with the given mixture representation.

⁴This is fortunate, because the mixture of reflected random walks we used for the numerical examination of entry and exit thresholds in Sections 4 and 5 does not satisfy Assumption 2.

and where for $k = 1, \dots, K$,

$$Q_k^K(c|C) = \begin{cases} 0, & \text{if } c - \mu_k^K(C) < -\sigma_k^K/2, \\ (c - \mu_k^K(C) + \sigma_k^K/2)/\sigma_k^K, & \text{if } -\sigma_k^K/2 \leq c - \mu_k^K(C) < \sigma_k^K/2, \\ 1, & \text{otherwise,} \end{cases}$$

$$\sigma_k^K \geq 0,$$

$$\mu_k^K(C) \text{ weakly increases in } C,$$

$$\hat{C} + \sigma_k^K/2 \leq \mu_k^K(C) \leq \check{C} - \sigma_k^K/2,$$

$$\mu_k^K(C) \leq C + \sigma_k^K/2,$$

then firms with all ranks follow threshold rules.

2. THE STOCHASTIC MONOTONICITY EXAMPLE

As discussed in our original article, the assumption that $Q(\cdot|C)$ weakly decreases with C —stochastic monotonicity—guarantees that a threshold-based rule governs the entry and exit of a firm that faces no possibility of competition. We used an example to demonstrate that this condition does *not* guarantee that oligopolists use thresholds. While the example itself was correct, we incorrectly stated that its stochastic demand process satisfied Assumptions 1 and 2. In fact, the example given does not satisfy Assumption 2. Here, we provide an alternative example that does satisfy both assumptions.

Set the maximum number of operational firms $\check{N} = 2$ and let C_t take values in $[0, 3]$. Suppose that

$$C' = \mu(C) + U'$$

where $\Pr[U' = -0.3|C] = \Pr[U' = 0.3|C] = 1/2$ and

$$\mu(C) = \begin{cases} 0.3 & \text{if } C \leq 0.3, \\ C & \text{if } 0.3 < C < 2.7, \\ 2.7 & \text{if } C \geq 2.7. \end{cases}$$

This displays stochastic monotonicity and satisfies Assumptions 1 and 2. The model's other parameters in this example are $\varphi(1) = \varphi(2) = 10$, $\pi(N) = 2 \times I\{N \leq 2\}$, $\kappa = 1$, and $\beta = 1.05^{-1}$. Figure 1 shows the value functions for the first and second entrants in its top and bottom panels, respectively.⁵ Clearly, the equilibrium entry rule for a firm with prospective rank 1 does not obey a threshold rule. Such a firm will enter in the disconnected sets \mathcal{A} and \mathcal{B} , where the firm's value exceeds the cost of entry.

⁵For the computation, we restricted C_t to a grid of 3001 evenly spaced points and used the algorithm described in Section 4 of the original article.

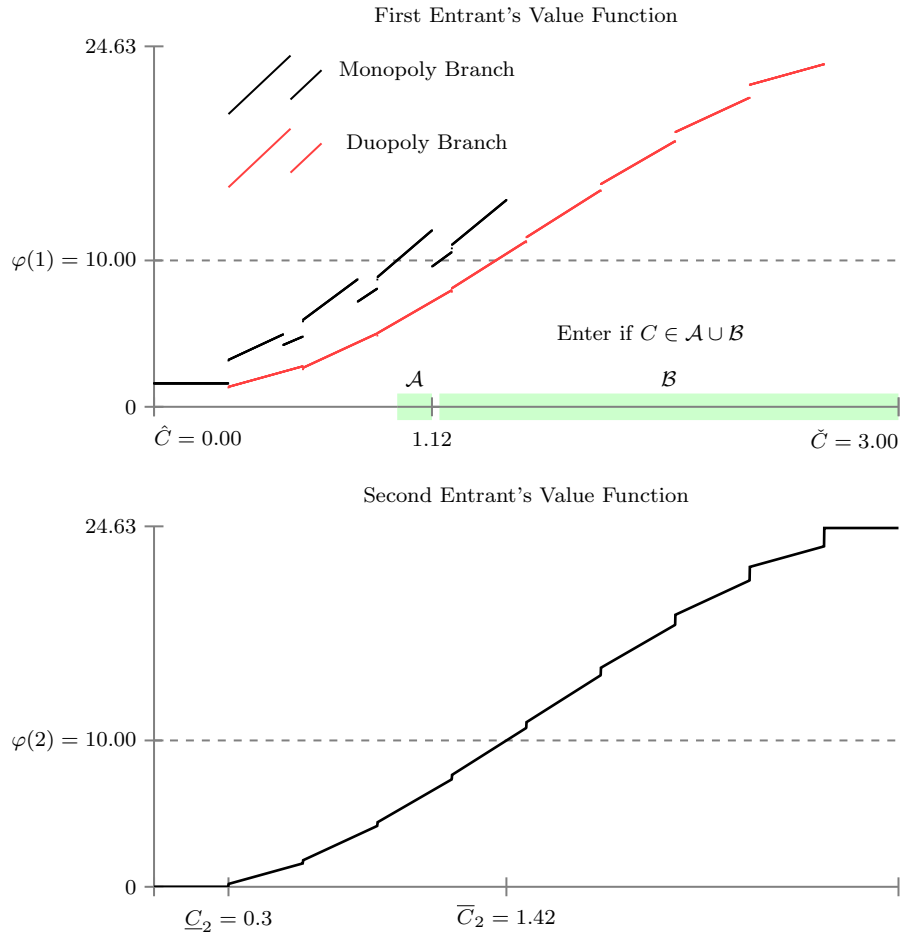


FIGURE 1.— Example of LIFO equilibrium with a nonmonotonic entry rule.

In this example, a firm with the opportunity to become the second entrant takes it if $C_t > 1.42$. The value of entry for a first entrant exceeds its cost if C_t is slightly less than $1.42 - 0.30 = 1.12$. Increasing C_t from such a value pushes the highest possible realization of C_{t+1} past 1.42. This discontinuously lowers the first entrant's value to a point below the cost of entry. Hence, the equilibrium entry strategy is not monotonic in C_t . Please see the text for further details.

*CentER, Dept. of Econometrics and Operations Research, Tilburg University,
P.O. Box 90153, 5000 LE Tilburg, The Netherlands; jaap@abbring.org*

and

*Federal Reserve Bank of Chicago, 230 South LaSalle Street, Chicago, IL 60604,
U.S.A.; jcampbell@frbchi.org.*

APPENDIX A: PROOF OF THE CORRECTED PROPOSITION

PROOF OF PROPOSITION 5*: The second part of Abbring and Campbell's proof establishes that Assumptions 1–3 imply Proposition 5*'s mixture representation. Specifically, it constructs a mixture representation that satisfies all the conditions in Proposition 5*, including the new restriction (1*). (The original proof set $\mu_k^K(C)$ in the mixture representation to $\nu_k^K + \mathbb{E}[C'|C]$; where ν_k^K is a constant that depends only on \underline{U} and the distribution of U' .) It remains to prove the converse implication.

Suppose that $Q(\cdot|C)$ has Proposition 5*'s mixture representation. We adapt Steps (i)–(iii) of the first part of Abbring and Campbell's proof to show that $Q(\cdot|C)$ satisfies Assumptions 1–3. Although the original proof's error was isolated to the demonstration that the expectational error U' is independent of C , we repeat all three steps of this proof for completeness. Nevertheless, this proof differs somewhat from the original because (1*) allows us to shorten its third step.

- (i). Assumption 1 is satisfied, because $\mu_k^K(C)$ is nondecreasing with C for all k, K . Since $\mu_k^K(C) \leq \bar{C}$ (as defined in the text), $\mathbb{E}[C'|C]$ equals $\mu(C) \equiv \lim_{K \rightarrow \infty} \sum_{k=1}^K p_k^K \mu_k^K(C)$ by the bounded convergence theorem (Billingsley, 1995, Theorem 16.5); and the set of nondecreasing functions is closed.
- (ii). Define $\tilde{Q}_k^K(u|C) \equiv Q_k^K(u + \mu(C)|C)$ and denote the distribution of U' given C with $\tilde{Q}(u|C) \equiv Q(u + \mu(C)|C)$. Because $Q(\cdot|C) = \lim_{K \rightarrow \infty} \sum_{k=1}^K p_k^K Q_k^K(\cdot|C)$,

$$(2^*) \quad \tilde{Q}(\cdot|C) = \lim_{K \rightarrow \infty} \sum_{k=1}^K p_k^K \tilde{Q}_k^K(\cdot|C).$$

Moreover, because $\mu(C) - \nu(C)$ does not depend on C , the functions $\tilde{Q}_k^K(\cdot|C)$, $k = 1, \dots, K$, $K \in \mathbb{N}$, do not depend on C . Therefore, $\tilde{Q}(\cdot|C)$ does not depend on C and $Q(\cdot|C)$ satisfies Assumption 2.

- (iii). Subtracting $\mu(C)$ from both sides of the proposition's penultimate condition and rearranging, we obtain

$$\mu_k^K(C) - \sigma_k^K/2 - \mu(C) \leq C - \mu(C).$$

Consequently, the support of $\tilde{Q}_k^K(\cdot|C)$ either includes or lies below $C - \mu(C)$. Since this distribution is also uniform, a result of A.I. Khintchine (Feller, 1971, p. 158) implies that $\tilde{Q}(\cdot|C)$ is concave on $[C - \mu(C), \infty)$. We have already shown that Assumption 2 holds good, so that $\tilde{Q}(\cdot|C)$ equals $\tilde{Q}(\cdot)$, and is concave above $C - \mu(C)$ for all C . Because $\mu(C) = \mathbb{E}[C'|C]$, this implies Assumption 3.

Q.E.D.

REFERENCES

- ABBRING, J. H. AND J. R. CAMPBELL (2010): "Last-In First-Out Oligopoly Dynamics," *Econometrica*, 78, 1491–1527. 1, 6
- BILLINGSLEY, P. (1995): *Probability and Measure*, New York: Wiley, third ed. 6
- FELLER, W. (1971): *An Introduction to Probability Theory and Its Applications*, vol. II, New York, NY: Wiley, 2nd ed. 6