Comments on 'On the internal structure of cities'

Xiaofang Dong[*,1] and Stephen L. Ross[**,2]

Abstract

This note documents a convergence problems with the simulations presented in Lucas and Rossi-Hansberg (2002). The simulation problem arises from the use of polar coordinates for integral approximation because the density of rays in a given area falls as the radius of the circle increases. Their simulations suggest that rents rise rapidly as the center of the city is approached, but in fact rents are either flat or fall as the center is approached.

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Lucas and Rossi-Hansberg (2002), hence forth referenced as L&RH, develop spatial models of cities in which economic activity self-organizes around itself due to spillovers in production.[1] However, there is a striking inconsistency between the spatial pattern of rents presented in the paper and the economic incentives faced by firms. As shown in Figure 1, the simulated rents increase quickly as the center of the city is approached, and rents can only behave this way in equilibrium if the returns to agglomeration also increase quickly near the center. The return to agglomeration depends upon exposure to other firms, and near the center of a symmetric city small changes in firm location should have almost no impact on the exposure of firms to other firms. Therefore, firms near the center of the city should be unwilling to pay the large rent premium for moving closer to the center that is implied by the simulated rents presented in L&RH.[2]

This problem arises because of a precision error in the simulations conducted by L&RH. Their simulations are conducted in polar coordinates using a fixed number of rays for approximating returns to agglomeration, and as one moves further from the center of the city the rays provide less and less dense coverage of the area surrounding any firm location. This precision error leads to increasing rents and density as the center is approached and causes economic activity to be heavily concentrated towards the center of the city.[3]

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[1] Also see work by Fujita and Ogawa (1989).

[2] This problem also arises in the simulations presented in Lucas (1999).

[3] When L&RH was written, the computing power necessary for approximating integrals over the very dense grids required for convergence was not easily available. In revised simulations provided by Professors Lucas and Rossi-Hansberg for Lucas (1999), the central peak remained using 1000 rays and 1000 points. Their model converged using 10,000 rays and 10,000 points, which required more than a day on a state of the art computer.

In Dong and Ross (2015), we prove that firm rents and densities fall as the center of the city is approached for monocentric equilibria where firms win the bid for land in the city center, and the density of employment is significantly more dispersed for the monocentric equilibria than in the original simulations in L&RH, see Figure 2. We also replicate all of the simulations in L&RH using two approaches: calculating the firm density over a rectangular grid, and calculating over polar coordinates using a recursive adaptive Simpson quadrature approach. The simulations converge relatively quickly, e.g. using a few hundred points in each dimension.

In summary, the problems with the figures in L&RH relate only to errors caused by imprecision in the simulations. The proof of existence and the strategy for identifying equilibria presented in their paper are both correct, and the qualitative land use patterns illustrated by their simulations are relatively unchanged in the revised simulations (Dong and Ross 2005): a traditional monocentric equilibria for low commuting costs, and increasing prevalence of mixed land use as commuting costs rise. However, for all land use patterns, the corrected simulations show lower rents and densities near the center of the city than implied by the L&RH simulations. Computer programs for conducting the simulations are available on the Journal of Economic Geography website.

References

Dong, X. Ross S.L. (2015) Accuracy and Efficiency in Simulating Equilibrium Land Use Patterns for Self-Organizing Cities, *Journal of Economic Geography*, 15, 707-722

Fujita, M., Ogawa H. (1989) Nonmonocentric urban configurations in a two-dimensional space, *Environment and planning A*, 21: 363-374.

Lucas, R. (1999) Externalities and cities, *Review of Economic Dynamics*, 4: 245-274.

Lucas, R., Rossi-Hansberg, E. (2002) On the internal structure of cities, *Econometrica*, 70: 1445-76.

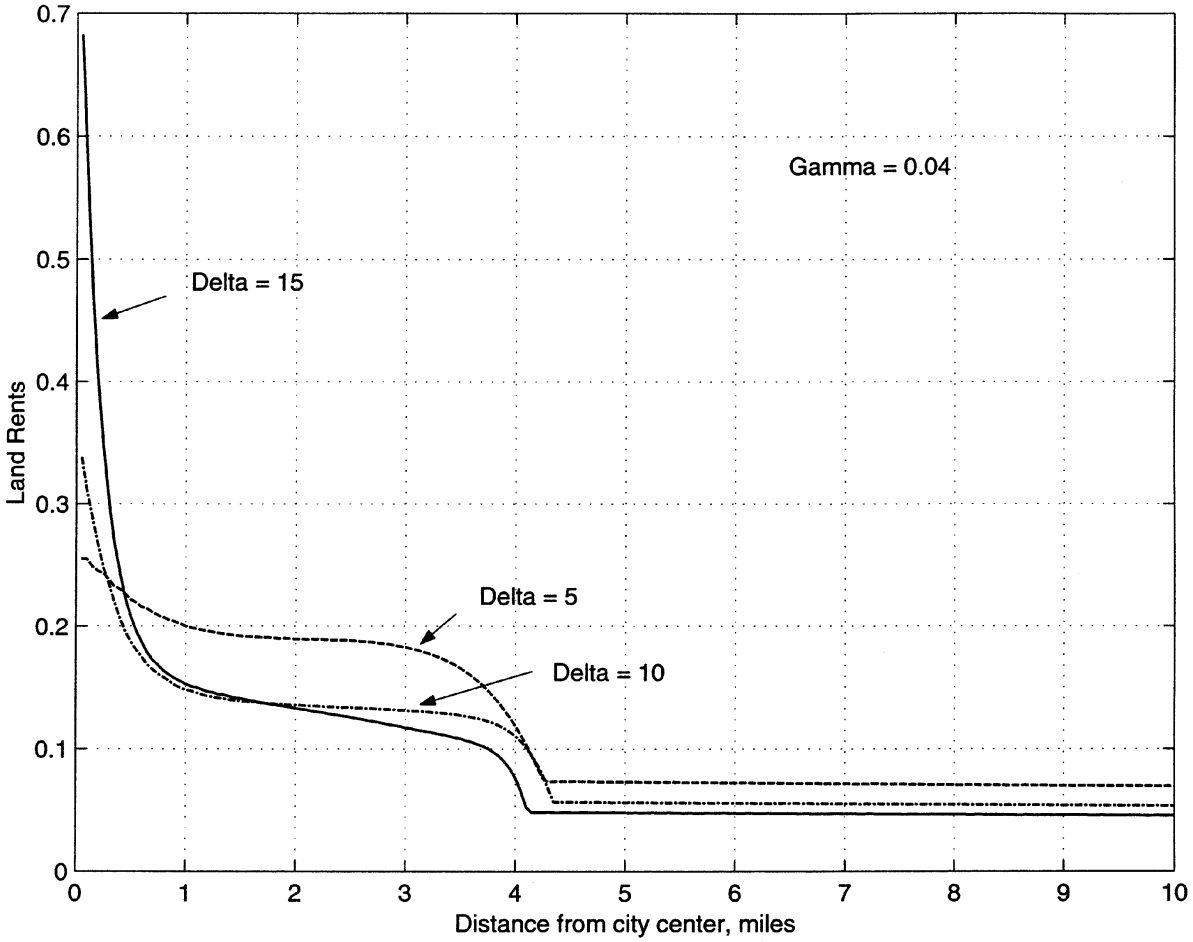

Figure 1. Reproduced from Figure 9 Land rents, various delta values, kappa = 0.001 in Lucas and Rossi-Hansberg (p. 1467, 2002).

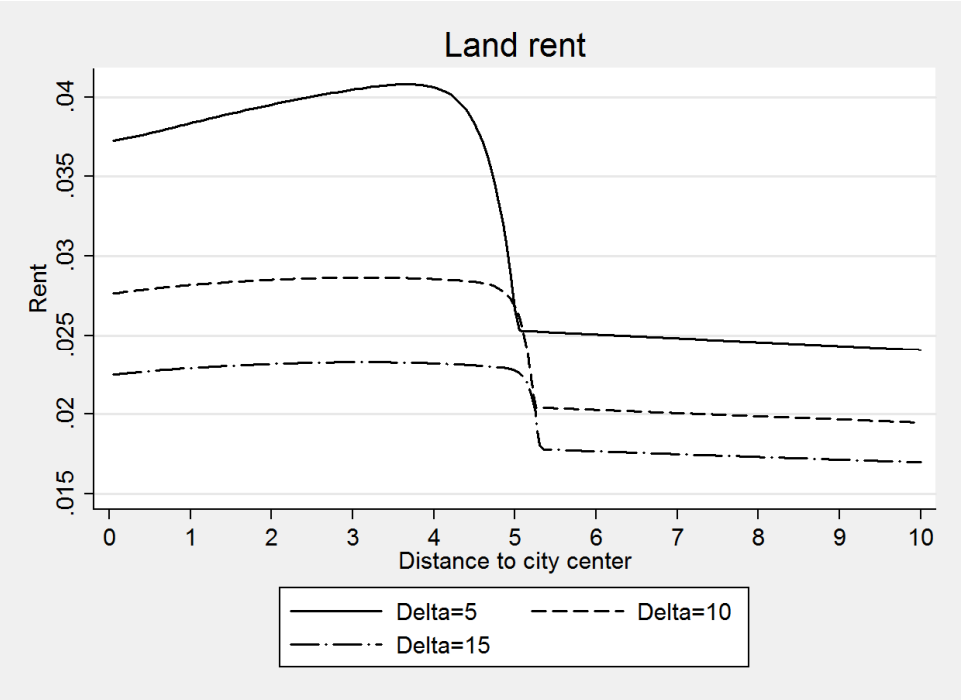

Figure 2. Reproduced from Figure 3 Monocentric Land Use Pattern using a Rectangular Grid in Dong and Ross (2015).

Notes. Simulations conducted using a rectangular grid with 200 by 200 points using the same parameter values as used in Figure 1.